LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2023**

UMT 1501 – ALGEBRA

Date: 01-11-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)
	Answer ALL the Questions - **(10 x 1 = 10)**
1.	**Answer the following**
a)	Write the other roots of the biquadratic equation when $\sqrt{5}+\sqrt{2}$ is one of its roots.
b)	When will be an equation f(x) = 0 is known as complete?
c)	State Exponential theorem.
d)	What is the characteristic equation of the matrix A?
e)	Find the number of integers less than and prime to 729?
2.	**Fill in the blanks**
a)	In an equation with real coefficients, _____ roots occur in pairs.
b)	Using _____ rule, one can ascertain whether an equation f(x) = 0 has imaginary roots or not.
c)	If $-1 < x < 1$, then $log\ (1+x)$ _____.
d)	Every square matrix satisfies it's _____.
e)	The sum of the integers less than N and prime to it including unity is _____.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Choose the correct answers for the following**
a)	Write the roots of the cubic equation which are in Arithmetical progression. i) a/d, a, a×d ii) a-d ,a, a + d iii) a + d, a/d, a-d iv) None of these
b)	The series of signs of the terms in the polynomial $x^7+8x^5-x+9=0$ is i) + + - + ii) + - - + iii) - + - + iv) + + - -
c)	Sum to infinity of the series $1+\frac{x}{1!}\log_e a+\frac{x^2}{2!}(\log_e a)^2+...+\frac{x^r}{r!}(\log_e a)^r+...$, (where a is a positive number) = i) x^a ii) e^a iii) a^x iv) log(a + x)
d)	The sum of the elements on the diagonal A is i) The product of the eigenvalues of A ii) The sum of the eigenvalues of A iii) determinant of A iv) None of these
e)	The number of integers less than and prime to 720 is i) 192 ii) 172 iii) 182 iv) 162
4.	**State True or False**
a)	Every equation f(x) = 0 of the n^{th} degree has n roots and no more.
b)	Cardon's method is used to determine both commensurable and the incommensurable roots of a numerical equation.
c)	The coefficient of x^n in the infinite series $1+\frac{b+ax}{1!}+\frac{(b+ax)^2}{2!}+...+\frac{(b+ax)^n}{n!}+...$ is $\frac{e^b a^n}{n!}$.

d)	The determinant of the matrix A is equal to the product of its eigen values.
e)	Fermat's theorem states that if p is a prime and a is any number prime to p then $a^{p-1}-1$ is not divisible by p.
	SECTION B - K3 (CO2)
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Solve the equation $x^4-5x^3+4x^2+8x-8=0$ given that one of the root is $1-\sqrt{5}$.
6.	Solve the equation $81x^3-18x^2-36x+8=0$ whose roots are in harmonic progression. Also write the roots of the cubic polynomial in GP and AP.
7.	Sum to infinity of the series $\frac{11.14}{10.15.20}+\frac{11.14.17}{10.15.20.25}+\ldots.$
8.	Determine the Characteristic equation of the matrix A=$\begin{bmatrix}2&2&0\\2&1&1\\-7&2&-3\end{bmatrix}$ and hence find its inverse.
	SECTION C – K4 (CO3)
Answer any TWO of the following	**(2 x 10 = 20)**
9.	If the sum of two roots of the equation $x^4+px^3+qx^2+rx+s=0$ equals the sum of the other two, show that $p^3+8r=4pq$.
10.	Predict the roots of the reciprocal equation $6x^5-x^4-43x^3+43x^2+x-6=0$.
11.	Verify Cayley Hamilton theorem for the matrix A=$\begin{bmatrix}1&-1&2\\-2&1&3\\3&2&-3\end{bmatrix}$.
12.	Evaluate the remainder obtained in dividing 2^{46} by 47.
	SECTION D – K5 (CO4)
Answer any ONE of the following	**(1 x 20 = 20)**
13.	a) Find the condition that the roots of the equation $ax^3+3bx^2+3cx+d=0$ may be geometric progression. Also solve $27x^3+42x^2-28x-8=0$ whose roots are in GP. (15 Marks) b) Determine the nature of the roots of the equation $x^5-6x^2-4x+5=0$. (5 Marks)
14.	a) Sum the series $\frac{5}{1!}+\frac{7}{3!}+\frac{9}{5}+\ldots$ (8 Marks) b) Determine the eigen values and eigen vectors of $\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}$. (12 Marks)
	SECTION E – K6 (CO5)
Answer any ONE of the following	**(1 x 20 = 20)**
15.	a) Estimate the sum of the fourth powers of the roots of $x^7+5x^4+1=0$ using Newton's theorem. . (5 Marks) b) Predict the positive root of the equation $x^3-2x^2-3x-4=0$ correct to 2 decimals using Horner's method. (15 Marks)
16.	a) Resolve into Partial fractions $\frac{9}{(x-1)(x+2)^2}$. (8 Marks) b) Apply Cardon's method to solve the equation $x^3-6x-9=0$. (12 Marks)

&&&&&&&&&&